NAG Toolbox for MATLAB

g02gk

1 Purpose

g02gk calculates the estimates of the parameters of a generalized linear model for given constraints from the singular value decomposition results.

2 Syntax

```
[b, se, cov, ifail] = g02gk(v, c, b, s, 'ip', ip, 'iconst', iconst)
```

3 Description

g02gk computes the estimates given a set of linear constraints for a generalized linear model which is not of full rank. It is intended for use after a call to g02ga, g02gb, g02gc or g02gd.

In the case of a model not of full rank the functions use a singular value decomposition to find the parameter estimates, $\hat{\beta}_{\mathrm{svd}}$, and their variance-covariance matrix. Details of the SVD are made available in the form of the matrix P^*:

$$P^* = \begin{pmatrix} D^{-1}P_1^{\mathrm{T}} \\ P_0^{\mathrm{T}} \end{pmatrix}$$

as described by g02ga, g02gb, g02gc and g02gd. Alternative solutions can be formed by imposing constraints on the parameters. If there are p parameters and the rank of the model is k then $n_{\mathrm{c}} = p - k$ constraints will have to be imposed to obtain a unique solution.

Let C be a p by n_{c} matrix of constraints, such that

$$C^{\mathrm{T}}\beta = 0,$$

then the new parameter estimates $\hat{\beta}_{\mathrm{c}}$ are given by:

$$\begin{aligned} \hat{\beta}_{\mathrm{c}} &= A\hat{\beta}_{\mathrm{svd}} \\ &= \left(I - P_0\left(C^{\mathrm{T}}P_0\right)^{-1}\right)\hat{\beta}_{\mathrm{svd}}, \qquad \text{where } I \text{ is the identity matrix,} \end{aligned}$$

and the variance-covariance matrix is given by

$$AP_1D^{-2}P_1^{\mathrm{T}}A^{\mathrm{T}}$$

provided $\left(C^{\mathrm{T}}P_0\right)^{-1}$ exists.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

McCullagh P and Nelder J A 1983 *Generalized Linear Models* Chapman and Hall

Searle S R 1971 *Linear Models* Wiley

5 Parameters

5.1 Compulsory Input Parameters

1: **v(ldv,ip + 7) – double array**

ldv, the first dimension of the array, must be at least **ip**. **ldv** should be as supplied to g02ga, g02gb, g02gc or g02gd.

The array **v** as returned by g02ga, g02gb, g02gc or g02gd.

2: **c**(**ldc**,**iconst**) – **double array**

ldc, the first dimension of the array, must be at least **ip**.

Contains the **iconst** constraints stored by column, i.e., the ith constraint is stored in the ith column of **c**.

3: **b**(**ip**) – **double array**

The parameter estimates computed by using the singular value decomposition, $\hat{\beta}_{\mathrm{svd}}$.

4: **s** – **double scalar**

The estimate of the scale parameter.

For results from g02ga and g02gd then **s** is the scale parameter for the model, σ^2.

For results from g02gb and g02gc then **s** should be set to 1.0.

Constraint: $\mathbf{s} > 0.0$.

5.2 Optional Input Parameters

1: **ip** – **int32 scalar**

Default: The dimension of the arrays **p**, **ip**, **se**, **cov**. (An error is raised if these dimensions are not equal.)

p, the number of terms in the linear model.

Constraint: $\mathbf{ip} \geq 1$.

2: **iconst** – **int32 scalar**

Default: The dimension of the array **c**.

the number of constraints to be imposed on the parameters, n_{c}.

Constraint: $0 < \mathbf{iconst} < \mathbf{ip}$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldv, ldc, wk

5.4 Output Parameters

1: **b**(**ip**) – **double array**

The parameter estimates of the parameters with the constraints imposed, $\hat{\beta}_{\mathrm{c}}$.

2: **se**(**ip**) – **double array**

The standard error of the parameter estimates in **b**.

3: **cov**((**ip** $\times$ (**ip** + **1**)/**2**)) – **double array**

The upper triangular part of the variance-covariance matrix of the **ip** parameter estimates given in **b**. They are stored packed by column, i.e., the covariance between the parameter estimate given in $\mathbf{b}(i)$ and the parameter estimate given in $\mathbf{b}(j)$, $j \geq i$, is stored in $\mathbf{cov}(j \times (j-1)/2 + i)$.

4: **ifail** – **int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **ip** < 1.
or **iconst** $\geq$ **ip**,
or **iconst** ≤ 0,
or **ldv** $<$ **ip**,
or **ldc** $<$ **ip**,
or **s** ≤ 0.0.

ifail $= 2$

c does not give a model of full rank.

7 Accuracy

It should be noted that due to rounding errors a parameter that should be zero when the constraints have been imposed may be returned as a value of order ***machine precision***.

8 Further Comments

g02gk is intended for use in situations in which dummy $(0 - -1)$ variables have been used such as in the analysis of designed experiments when you do not wish to change the parameters of the model to give a full rank model. The function is not intended for situations in which the relationships between the independent variables are only approximate.

9 Example

```
v    =    [4.890297476493481,    132.9931304975522,    0.08671323636775569,
11.53226476012202, ...
        0.6875039713237874,  0.6035396163882434,  0,  0.0190106784478845,
0.008405335397184037, ...
       0.008587821997748737, 0.002017521052951722, 0.006018698150011753,
...
       0.002569439508216107,  0.005710268118949224,  0.003195800397035001,
0.001516472273672414;
             4.150630280659896,   63.47399412403458,   0.1255168658332746,
7.96705680436851, ...
     0.4385677120269487, 0.5137644803624514, 0, -0.000210405285073920 8, -
0.03362644601906961, ...
     0.03352834001769896, -0.0001122992837032618, -8.410523978672018e-05,
...
      -1.834202533241677e-05, -7.350180559025446e-05, -2.50373181392074e-
05, -9.418896225321663e-06;
            4.847173050021614,   127.3797841010689,   0.08860326907783028,
11.28626528578293, ...
     -1.207211262022778, 0.5962906923855547, 0, -0.000593266263451917, -
0.0003445349204235381, ...
       -0.000333175581116977, 8.444423808860036e-05, 0.04164067761137802,
...
    -0.000834602710139113, -0.03974432490123243, -0.001281738894244829, -
0.0003732773692135418;
             4.347583499449088,   77.29146221158076,   0.1137455037136346,
8.791556302019613, ...
      0.193629026736895, 0.5316079856446033, 0, -0.005604448606553034, -
0.00473076223435367, ...
       -0.004660408745794929, 0.003786722373595579, 0.03079195529372083,
...
      -0.01961182893501804, 0.03434336417204345, -0.04455401443460466, -
```

```
0.006573924702694605;
              3.660007365784039,   38.86162911664574,   0.160412976990412,
6.23390961729842, ...
       0.02218333268101244,  0.4819807366090336,  0,  -0.01246501079735563,
0.02562719062269757, ...
        0.02538801446619446,  -0.06348021588624767,  -0.003268009116713865,
...
       0.01218311975746848,  -0.003443738617825787,  -0.01974708083814387,
0.001810698017859427;
             4.906081344202832,   135.1089303020151,   0.08603159451608589,
11.62363670724507, ...
      -0.3553126814370362,  0.6083327470988832,  0,  -0.007494797532342946,
0.006087596749368498, ...
        0.006034528387724169,  -0.01961692266943561,  0.009789429893167587,
...
       -0.06230084141169696,  0.01026746382279108,  0.04102802603588968,  -
0.006278875872494353;
              4.166414148369245,   64.48380766743846,   0.1245301935277972,
8.030181048235367, ...
      0.1880789664600944,  0.5196429754104227,  0,  -0.007856295692343652,  -
0.001520639263699578, ...
       -0.001513684000176248,  -0.004821972428467818,  0.01450100243772308,
...
        0.03053649776467425,  0.01481400539841174,  0.02215701246445583,  -
0.08986481375760852;
             4.862956917730965,   129.4062806673598,   0.08790676991659417,
11.37568814038781, ...
        1.174924303092439,  0.6011714612100958,  0,  0.4644148349507297,  -
0.508180337140321, ...
      -0.508180337140321,  -0.508180337140321,  0.04376550218959127, ...
          0.04376550218959111,   0.04376550218959127,   0.04376550218959124,
0.04376550218959125;
              4.363367367158438,   78.52109911103648,   0.1128513646061788,
8.861213185057478, ...
       -0.7464706890225739,  0.5372707561039687,  0,  -0.3635174659273034,  -
0.05120849696428324, ...
      -0.05120849696428324,  -0.05120849696428316,  0.4147259628915867, ...
            0.4147259628915865,   0.4147259628915868,   0.4147259628915868,
0.4147259628915868];
c = [0, 0;
     1, 0;
     1, 0;
     1, 0;
     0, 1;
     0, 1;
     0, 1;
     0, 1;
     0, 1];
b = [2.597657842414576;
     1.261948923584132;
     1.277732791293482;
     0.05797612753696259;
     1.030690710494773;
     0.2910235146611871;
     0.9875662840229057;
     0.4879767334503795;
     -0.1995994002146691];
s = 1;
[bOut, se, cov, ifail] = g02gk(v, c, b, s)
```

```
bOut =
    3.9831
    0.3961
    0.4118
   -0.8079
    0.5112
   -0.2285
    0.4680
   -0.0316
   -0.7191
```

```
se =
    0.0396
    0.0458
    0.0457
    0.0622
    0.0562
    0.0727
    0.0569
    0.0675
    0.0887
cov =
    0.0016
   -0.0006
    0.0021
   -0.0006
   -0.0002
    0.0021
    0.0012
   -0.0019
   -0.0019
    0.0039
   -0.0006
    0.0000
   -0.0000
    0.0000
    0.0032
    0.0002
   -0.0000
    0.0000
   -0.0000
   -0.0008
    0.0053
   -0.0005
    0.0000
   -0.0000
   -0.0000
   -0.0001
   -0.0008
    0.0032
   -0.0001
    0.0000
   -0.0000
   -0.0000
   -0.0006
   -0.0013
   -0.0006
    0.0046
    0.0010
   -0.0000
    0.0000
    0.0000
   -0.0017
   -0.0024
   -0.0017
   -0.0021
    0.0079
ifail =
           0
```